Digital Systems and Information Representation

Note Title 9/5/2007

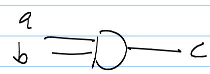
Base signals two-valued 0 or 1

"bit" smallest unambiguous unit of information

propositional calc.

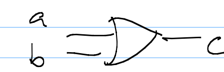
operations

AND $a \cdot b = c$



c is true iff a is true and b is true

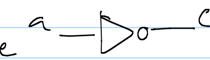
OR $a + b = c$



c iff a or b or both

NOT $a' = c$

$\bar{a} = c$ a is true iff c is false



$\sim a = c$

$\neg a = c$

e.g.

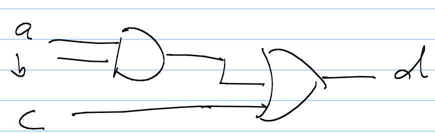
a passed microeconomics

b " macro "

c " econ survey

d met econ grad. req.

$d = a \cdot b + c$



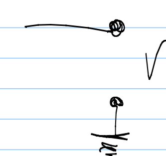
1 true

0 false

Boolean algebra

physical representation

Wire



$V > V_H \Rightarrow 1$

$V < V_L \Rightarrow 0$

$V_H > V_L$

region between V_L and V_H is undefined

two values are not enough

→ numbers, text, images, audio, video, ...

sequence of number systems

1. "counting" numbers $1, 2, 3, \ldots$ (positive integers)
2. what if I don't have any stuff?
   - add 0 as a number

- what if I take away more than I have?

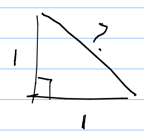
  new concept: negative numbers

  can solve equation $x + a = 0$ integer a

why can't I divide my stuff 3 ways?

incorporate rational numbers

eqn $ax + b = 0$ integers a, b if $a \neq 0$



how long is this line?

expand to irrational numbers

and get "reals"

$x^2 - 2 = 0$

what about $x^2 + 2 = 0$

complex numbers

vector number system with 2 components

(a, b) a, b both "real" numbers

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rules

equality $(a, b) = (c, d)$

iff $a = c$ and $b = d$

add $(a, b) + (c, d) = (a + c, b + d)$

mult. 2 complex #s

$(a, b) \times (c, d) = (ac - bd, ad + bc)$

scalar and complex

$c \times (a, b) = (ac, bc)$

complx #s w/ 2^{nd} comp 0 have same properties as reals

$$(a, 0) = (c, 0) \text{ iff } a = c$$

$$(a, 0) + (c, 0) = (a+c, 0)$$

$$(a, 0) \times (c, 0) = (ac, 0)$$

$$(a, 0) \times c = (ac, 0)$$

1 st comp. of complex is called "real"

then follow historical naming conventions

rational → irrational

real → imaginary

2^{nd} component of complex # known as imaginary component

equ. $x^2 + 2 = 0$

$$x^2 + (2, 0) = (0, 0)$$

insert $x = (0, \sqrt{2})$

$$x^2 + (2, 0) = (0, 0)$$

$$(0, \sqrt{2}) \times (0, \sqrt{2}) + (2, 0) = (0, 0)$$

$$(-2, 0) + (2, 0) = (0, 0) \quad \checkmark$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = \sqrt{-1}$$

$$x = (0, 1)$$

$$(0, 1)^2 + (1, 0) = (0, 0)$$

$$(-1, 0) + (1, 0) = (0, 0) \quad \checkmark$$

$$x = \sqrt{-1} = (0, 1)$$

new symbol $i \equiv \sqrt{-1} = (0, 1)$

(a, b) can now be written $a + ib$

$(a, b) = a + ib \qquad i = (0,1)$

$a + ib = (a, 0) + (0,1) \times (b, 0)$

$= (a, 0) + (0, b)$

$= (a, b)$

EEs use j instead of i

can now solve

$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} \cdots a_{n-1} x + a_n = 0$

if a's comply, $n \geq 1$ and $a_0 \neq 0 \Rightarrow n$ roots

Back to representing signal (numbers) that are multi-valued

positional number systems

base 10 $\quad xyz_{10} = x \cdot 10^2 + y \cdot 10^1 + z \cdot 10^0$

x, y, z have values 0 to 9

base 2 $\quad xyz_2 = x \cdot 2^2 + y \cdot 2^1 + z \cdot 2^0$

x, y, z value values 0 or 1

x y z	
0 0 0	0
0 0 1	1
0 1 0	2
0 1 1	3
1 0 0	4
1 0 1	5
1 1 0	6
1 1 1	7

base 2 called binary

with fixed # of bits one can rep negative numbers multiple ways

fixed # of digits (bits)

e.g. 4-bit binary # system

unsigned range 0 to 15

0000 to 1111

w/ n bits $\Rightarrow$ range 0 to $2^n - 1$

offset or bias (e.g. -7)

range -7 to $+8$

sign-magnitude

1st bit sign (0 = positive, 1 = negative)

bits 2,3,4 magnitude range 0 to 7

overall range -7 to 7

what about 1000? -0

w/ n bits $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

2's complement (radix complement)

$$wxyz = w\cdot -(2)^3 + x\cdot 2^2 + y\cdot 2^1 + z\cdot 2^0$$

$$= w\cdot(-8) + \cancel{x}\cdot 4\cdot y\cdot 2 + z$$

range is -8 to $+7$

1st bit still sign bit

only one zero 0000

positional # system

negative weight on msb

n bits

range $-(2^{n-1})$ to $+(2^{n-1}-1)$

only need adder for add/sub

2's complement is almost universal