Digital Systems and Information Representation

Basic signals: two-valued 0 or 1

"bit" smallest unambiguous unit of information

Boolean operations:
- AND: \( a \cdot b = c \)
- OR: \( a + b = c \)
- NOT: \( \overline{a} = c \)

Example:
- A passed microeconomics
- B: macro
- C: econ survey
- D: not econ grad, e.g.
  \[ d = a \cdot b + c \]

tram 0
false

Boolean algebra

Physical representation

\[ W: \begin{align*}
W_H &\rightarrow V \quad V > V_H \Rightarrow 1 \\
V_H &\rightarrow 0 \\
V_H &\rightarrow V_L
\end{align*} \]
region between \( V_0 \) and \( V_f \) is undefined

Two values are not enough.

Numbers, text, images, audio, video, ...

Sequence of number systems
1. "counting numbers" 1, 2, 3, ... (positive integers)
2. What if I don't have any stuff?
   - add 0 as a number

\[ x^2 - 2 = 0 \]

What about \( x^2 + 2 = 0 \)

complex numbers

vector number system with 2 components

\((a, b)\) \(a, b\) both "real" numbers

\[ \text{if } a = c \text{ and } b = d \]

add
\[ (a, b) + (c, d) = (a+c, b+d) \]

mult. 2 complex \#s
\[ (a, b) \times (c, d) = (ac - bd, ad + bc) \]

scalar \& complex
\[ c \times (a, b) = (ac, bc) \]
complex #s w/ 2nd comp 0 have same properties as reals

\[(a, 0) = (c, 0) \iff a = c\]
\[(a, 0) + (c, 0) = (a+c, 0)\]
\[(a, 0) \times (c, 0) = (ac, 0)\]
\[(a, 0) \times 1 = (a, 0)\]

1st comp of complex is called "real".
then follow historical naming conventions

Rational \rightarrow\text{ irrational}

Real \rightarrow\text{ imaginary}

2nd comp of complex # known as

imaginary component

e.g., \[x^2 + 2 = 0\]
\[x^2 + (2, 0) = (0, 0)\]

Insert \[x = (0, \sqrt{2})\]

\[x^2 + (2, 0) = (0, 0)\]
\[(0, \sqrt{2}) + (0, \sqrt{2}) \neq (2, 0) = (0, 0)\]
\[-(2, 0) + (2, 0) = (0, 0) \checkmark\]

\[x^2 + 1 = 0\]
\[x^2 = -1\]
\[x = \sqrt{-1} = (0, 1)\]

\[x^2 = (0, 1)\]
\[(0, 1)^2 + (1, 0) = (0, 0)\]
\[(-1, 0) + (1, 0) = (0, 0) \checkmark\]
\[x = \sqrt{-1} = (0, 1)\]

new symbol \(i \neq \sqrt{-1} = (0, 1)\)
\[(a, b)\ can\ now\ be\ written\ \ a + bi\]
\[(a, b) = a + ib \quad i = (0, 1)\]

\[a + ib = (a, 0) + (0, 1)k(b, 0) = (a, 0) + (0, b) = (a, b)\]

EEs use \(j\) instead of \(i\)

\[a_nx^n + a_{n-1}x^{n-1} + a_2x^2 + \ldots + a_0x^0 = 0\]

it's complex, \(n \geq 1\) and \(a_0 \neq 0 \Rightarrow n\) roots

Back to representing signal (numbers) that are multi-valued

**Positional Number Systems**

- **base 10**: \(xyz_a = x \cdot 10^2 + y \cdot 10^1 + z \cdot 10^0\)
  - \(x, y, z\) have values 0 to 9
- **base 2**: \(xyz_2 = x \cdot 2^2 + y \cdot 2^1 + z \cdot 2^0\)
  - \(x, y, z\) have values 0 or 1

**Fixed # of digits (bits)**

- **e.g. 4-bit binary # system**
  - **Unsigned**: range 0 to 15
    - \(0000 \rightarrow 0\) to \(1111 \rightarrow 15\)
  - \(n/\text{bits} \Rightarrow \text{range} 0 \text{ to } 2^n - 1\)
- \(\text{offset or bias (e.g. } -7)\)
  - \(\text{range } -2^{n-1} \text{ to } +2^{n-1}\)

*base 2 called binary*

with fixed # of bits one can rep negative numbers multiple ways
Sign - magnitude
1st bit sign \( (0 = \text{positive}, 1 = \text{negative}) \)
bits 2, 3, 4 magnitude range 0 to 7
overall range -7 to 7
what about 1000? -0
w/n bits \(-2^{n-1}\) to \(+2^{n-1}\)

2's complement (radix complement)
\[ w' \cdot y \cdot z = w \cdot x \cdot (2^3 + x \cdot 2^2 + y \cdot 2^1 + z^2) = w \cdot (-8) + x \cdot 4 + y \cdot 2 + z \]
range is -8 to 7
1st bit still sign bit
only one zero 0000
positioned # system
negative weight on mod

\[ n \text{ bits} \]
range \(-2^{n-1}\) to \(+2^{n-1}\)
only need adder for add/sub
2's complement is almost universal